higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1060**(E)**(M29)T

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

29 March 2017 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Questions must be answered in blue or black ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If z = $z = 3\ln(x - 2y) - e^{\frac{x}{y}}$ determine:

1.1.1 $\dfrac{\partial z}{\partial x}$ (1)

1.1.2 $\dfrac{\partial z}{\partial y}$ (1)

1.2 If the parametric equations of a curve are given as $y = 6 - 2t^3$ and $x = -3t + t^2$ find the equation of the tangent to the curve at the point where $t = -1$. (4)

[6]

QUESTION 2

Determine $\int y\ dx$ if:

2.1 $y = \cos^5 ax.\sin^3 ax$ (4)

2.2 $y = \dfrac{1}{\cot^4 \frac{x}{3}}$ (4)

2.3 $y = x^2.2^{3x}$ (4)

2.4 $y = \dfrac{1}{1 - x + 2x^2}$ (4)

2.5 $y = arc\cot x$ (2)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \dfrac{-x^2 + 3x + 1}{(1 - 2x)^3}\ dx$ (5)

3.2 $\int \dfrac{x^2 - 29x + 5}{(x - 4)(x^2 + 3)x}\ dx$ (7)

[12]

QUESTION 4

4.1 Calculate the particular solution of :

$$x\frac{dy}{dx} - y = x\ln x \quad \text{if} \quad y = 3 \text{ when } x = e$$ (5)

4.2 Calculate the particular solution of:

$$\left(\frac{1}{2}\right)\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 2x^2 \quad \text{if} \quad y = 1 \text{ when } x = 0 \text{ and } \frac{dy}{dx} = 1 \text{ when } x = 0$$ (7)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of the two curves $y = x^3$ and $y = x$.

Make a neat sketch of the two curves and show the area bounded by the curves in the first quadrant. Show the representative strip/element that you will use to calculate the volume (use the shell method only) of the solid generated when the area bounded by the curves rotates about the *y*-axis. (3)

5.1.2 Calculate the volume described in QUESTION 5.1.1. (4)

5.2 5.2.1 Calculate the points of intersection of $y = x^2$ and $y^2 = 27x$.

Make a neat sketch of the two curves and show the area bounded by the curves. Show the representative strip/element, perpendicular to the *x*-axis, that you will use to calculate the area bounded by the curves. (3)

5.2.2 Calculate the area described in QUESTION 5.2.1. (3)

5.2.3 Calculate the distance from the centroid to the *x*-axis of the bounded area described in QUESTION 5.2.1. (5)

5.3 5.3.1 Make a neat sketch of the graph $y = 3\sin x$ and show the representative strip/element that you will use to calculate the volume of the solid generated when the area bounded by the curve, the lines $x = 0$, $x = \frac{\pi}{4}$ and $y = 0$ rotates about the *x*-axis. (2)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (3)

5.3.3 Calculate the moment of inertia of the solid described in QUESTION 5.3.1. (5)

5.4 5.4.1 A vertical sluice gate in the form of a trapezium is 4 m high. The longest horizontal side is 6 m in length and 2 m below the water surface. The shortest side is 4 m in length and 6 m below the water surface.

Make a neat sketch of the sluice gate and show the representative strip/element that you will use to calculate the depth of the centre of pressure. Calculate the relation between the two variables *x* and *y*. (3)

5.4.2 Calculate, by using integration, the area moment of the sluice gate about the water surface. (4)

5.4.3 Calculate, by using integration, the second moment of area of the sluice gate about the water surface, as well as the depth of the centre of pressure on the sluice gate. (5)

[40]

QUESTION 6

6.1 Calculate the arc length of the curve given by the parametric equations, $x = 2e^t \sin t$ and $y = 2e^t \cos t$, over the interval $0 \le t \le \frac{\pi}{2}$. (7)

6.2 Calculate the surface area generated when the arc of $x = 3y^3$ is rotated about the *y*-axis between $y = 1$ and $y = 2$. (5)

[12]

TOTAL: 100

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \ \mu \ \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \ \mu \ \tan A \tan B}$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\operatorname{cosec} x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} \cdot \frac{d}{dx} ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} \cdot \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2} \right) \right] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\text{cosec}\, f(x)$	$-\text{cosec}\, f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln\ f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b} \sin^{-1} \frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b} \sin^{-1} \frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab} \ln \left(\frac{a + bx}{a - bx} \right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2} \ln \left[x + \sqrt{x^2 \pm b^2} \right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b} \ln \left[bx + \sqrt{b^2x^2 \pm a^2} \right] + C$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y\,dx \ ; \ A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy \ ; \ A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \,;\, V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \,;\, V_x = 2\pi \int_a^b xy dy$$

$$V_y = \pi \int_a^b x^2 dy \,;\, V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \,;\, V_y = 2\pi \int_a^b xy dx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \,;\, \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore ye^{\int Pdx} = \int Qe^{\int Pdx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$